

Problem 152 from chapter VI of Mahavira's "Ganitasarasangraha" ("Compendium of the Essence of Mathematics", 9th century)

"Pigeons are sold at the price of 5 for 3 panas, sarasa birds at the price of 7 for 5 panas, swans at the rate of 9 for 7 panas and peacocks at the price of 9 for 7 panas. A certain man was told to bring at these rates 100 birds for 100 panas for the amusement of the king's son and was sent to do so. What amount does he give for each kind of birds?"

Solution Let a , b , c , d be the number of pigeons, sarasa birds, swans and peacocks respectively, where a , b , c , d are positive integers each smaller than 100. Then we have the following system of Diophantine equations:

$$a + b + c + d = 100$$

$$\frac{3a}{5} + \frac{5b}{7} + \frac{7c}{9} + 3d = 100$$

Using the first equation we can eliminate a in the second equation and get:

$$9b + 14c + 189d = 3150 \quad (1)$$

from which follows that c is necessarily a multiple of 9 and b is a multiple of 7 and therefore

$$c = 9k \quad , \quad b = 7n \quad (2)$$

where k and n are positive integers, with $k < 11$ and $n < 14$. Then equation (1) yields:

$$3d = 50 - n - 2k \quad (3)$$

Then the number $50 - n - 2k$ is a multiple of 3 and we conclude that:

For $k = 1$ or 4 or 7 or 10 the possible values for n are 3, 6, 9, 12

For $k = 2$ or 5 or 8 the possible values for n are 1, 4, 7, 10, 13

For $k = 3$ or 6 or 9 the possible values for n are 2, 5, 8, 11

In the following table are calculated the corresponding values of b , c and d for each of these cases, using (1), (2) and (3). The black fields indicate the cases that are discarded because $b + c + d \geq 100$

		k										
n		1	2	3	4	5	6	7	8	9	10	
	1		7,18,15				7,45,13			7,72,11		
	2			14,27,14				14,54,12			14,81,10	
	3	21,9,15				21,36,13				21,63,11		21,90,9
	4		28,18,14				28,45,12			28,72,10		
	5			35,27,13				35,54,11			35,81,9	
	6	42,9,14				42,36,12				42,63,10		42,90,8
	7		49,18,13				49,45,11			49,72,9		
	8			56,27,12				56,54,10			56,81,8	
	9	63,9,13				63,36,11				63,63,9		63,90,7
	10		70,18,12				70,45,10			70,72,8		
	11			77,27,11				77,54,9			77,81,7	
	12	84,9,12				84,36,10				84,63,8		84,90,6
	13		91,18,11				91,45,9			91,72,7		

There are in total 16 solutions:

	pigeons	cost	sarasa	cost	swans	cost	peacocks	cost
1	60	36	7	5	18	14	15	45
2	35	21	7	5	45	35	13	39
3	10	6	7	5	72	56	11	33
4	45	27	14	10	27	21	14	42
5	20	12	14	10	54	42	12	36
6	55	33	21	15	9	7	15	45
7	30	18	21	15	36	28	13	39
8	5	3	21	15	63	49	11	33
9	40	24	28	20	18	14	14	42
10	15	9	28	20	45	35	12	36
11	25	15	35	25	27	21	13	39
12	35	21	42	30	9	7	14	42
13	10	6	42	30	36	28	12	36
14	20	12	49	35	18	14	13	39
15	5	3	56	40	27	21	12	36
16	15	9	63	45	9	7	13	39